Lösungen Gemischte Übungen:

zu a) $-\frac{5}{144}(3y-5)^3-\frac{1}{3}(4-\frac{1}{2}y)^2+\frac{221}{480}(5y-9)(5-2y)+\frac{13}{4}y^2=(1-y)^3+\frac{1}{4}y(\frac{1}{2}y+1)^2,\ G=\mathbb{Z}$

$$-\frac{5}{144}(3y-5)^3-\frac{1}{3}(4-\frac{1}{2}y)^2+\frac{221}{480}(5y-9)(5-2y)+\frac{13}{4}y^2=(1-y)^3+\frac{1}{4}y(\frac{1}{2}y+1)^2$$

$$\Leftrightarrow\quad -\frac{5}{144}(27y^3-135y^2+225y-125)-\frac{1}{3}(16-4y+\frac{1}{4}y^2)+\frac{221}{480}(25y-10y^2-45+18y)+\frac{13}{4}y^2=$$
$$=(1-3y+3y^2-y^3)+\frac{1}{4}y(\frac{1}{4}y^2+y+1)$$

$$\Leftrightarrow\quad -\frac{15}{16}y^3+\frac{75}{16}y^2-\frac{125}{16}y+\frac{625}{144}-\frac{16}{3}+\frac{4}{3}y-\frac{1}{12}y^2+\frac{1105}{96}y-\frac{221}{48}y^2-\frac{663}{32}+\frac{663}{80}y+\frac{13}{4}y^2= \quad |\cdot 1440$$
$$=1-3y+3y^2-y^3+\frac{1}{16}y^3+\frac{1}{4}y^2+\frac{1}{4}y$$

$$\Leftrightarrow\quad -1350y^3+6750y^2-11250y+6250-7680+1920y-120y^2+16575y-$$
$$-6630y^2-29835+11934y+4680y^2=$$
$$=1440-4320y+4320y^2-1440y^3+90y^3+360y^2+360y$$

$$\Leftrightarrow\quad -1350y^3+4680y^2+19179y-31265=-1350y^3+4680y^2-3960y+1440 \quad |+1350y^3-4680y^2$$

$$\Leftrightarrow\quad 19179y-31265=-3960y+1440 \quad |+3960y$$

$$\Leftrightarrow\quad 23139y-31265=1440 \quad |+31265$$

$$\Leftrightarrow\quad 23139y=32705 \quad |\div 23139$$

$$\Leftrightarrow\quad y=\frac{32705}{23139} \quad , \frac{32705}{23139}\notin\mathbb{Z}$$

$$\Rightarrow\quad L=\{\}$$

b) $-\frac{3}{2}[x-0{,}3(2x-7)^2-2x(3x+\frac{5}{2})^2-\frac{1}{5}]-(5x-2)^2=\frac{1}{4}(x+3)^2+27x^3+21{,}55x^2+7{,}05x \quad , \; G=\mathbb{Q}$

$$-\frac{3}{2}[x-0{,}3(2x-7)^2-2x(3x+\frac{5}{2})^2-\frac{1}{5}]-(5x-2)^2=\frac{1}{4}(x+3)^2+27x^3+21{,}55x^2+7{,}05x$$

$$\Leftrightarrow \quad -\frac{3}{2}[x-0{,}3(4x^2-28x+49)-2x(9x^2+15x+\frac{25}{4})-\frac{1}{5}]-(25x^2-20x+4)=$$
$$=\frac{1}{4}(x^2+6x+9)+27x^3+21{,}55x^2+7{,}05x$$

$$\Leftrightarrow \quad -\frac{3}{2}[x-1{,}2x^2+8{,}4x-14{,}7-18x^3-30x^2-12{,}5x-0{,}2]-25x^2+20x-4=$$
$$=\frac{1}{4}x^2+\frac{3}{2}x+\frac{9}{4}+27x^3+21{,}55x^2+7{,}05x$$

$$\Leftrightarrow \quad -\frac{3}{2}[-14{,}9-18x^3-31{,}2x^2-3{,}1x]-25x^2+20x-4=\frac{1}{4}x^2+\frac{3}{2}x+\frac{9}{4}+27x^3+21{,}55x^2+7{,}05x$$

$$\Leftrightarrow \quad 22{,}35+27x^3+46{,}8x^2+4{,}65x-25x^2+20x-4=0{,}25x^2+1{,}5x+2{,}25+27x^3+21{,}55x^2+7{,}05x$$

$$\Leftrightarrow \quad 27x^3+21{,}8x^2+24{,}65x+18{,}35=8{,}55x+2{,}25+27x^3+21{,}8x^2 \quad |-27x^3-21{,}8x^2$$

$$\Leftrightarrow \quad 24{,}65x+18{,}35=8{,}55x+2{,}25 \quad |-8{,}55x$$

$$\Leftrightarrow \quad 16{,}1x+18{,}35=2{,}25 \quad |-18{,}35$$

$$\Leftrightarrow \quad 16{,}1x=-16{,}1 \quad |\div 16{,}1$$

$$\Leftrightarrow \quad x=-1 \quad , \; -1\in\mathbb{Q}$$

$$\Rightarrow \quad L=\{-1\}$$

c) $(z-1)(2n-1)=n(z-n) \quad , \; z\in\mathbb{Q} \; \wedge \; n\in\mathbb{Z}$

$$(z-1)(2n-1)=n(z-n)$$

$$\Leftrightarrow \quad 2nz-z-2n+1=zn-n^2 \quad |-zn+2n-1$$

$$\Leftrightarrow \quad nz-z=2n-n^2-1$$

$$\Leftrightarrow \quad z(n-1)=-(n^2-2n+1)$$

$$\Leftrightarrow \quad z(n-1)=-(n-1)^2$$

1. Fall: $n=1$ $\Rightarrow$ $0\cdot z=0$ wahr $\forall\, z\in\mathbb{Q}$

$\Rightarrow$ $L=\mathbb{Q}$

2. Fall: $n\neq 1$ $\Rightarrow$ $z(n-1)=-(n-1)^2$ $\mid\div(n-1)$

$\Leftrightarrow$ $z=-(n-1)$

$\Leftrightarrow$ $z=1-n$

$\Rightarrow$ $L=\{1-n\mid n\in \setminus\{0\}\}$

d) $\dfrac{2x}{a+1}-\dfrac{2a-6}{a-1}=\dfrac{x}{a-1}$, $z\in\mathbb{Q}\wedge a\in IR\setminus\{-1;1\}$

$$\frac{2x}{a+1}-\frac{2a-6}{a-1}=\frac{x}{a-1} \qquad \mid\cdot(a+1)(a-1)$$

$\Leftrightarrow$ $2x(a-1)-(2a-6)(a+1)=x(a+1)$

$\Leftrightarrow$ $2ax-2x-(2a^2+2a-6a-6)=ax+x$

$\Leftrightarrow$ $2ax-2x-2a^2+4a+6=ax+x$ $\mid -ax-x+2a^2-4a-6$

$\Leftrightarrow$ $ax-3x=2a^2-4a-6$

$\Leftrightarrow$ $x(a-3)=2(a^2-2a-3)$

$\Leftrightarrow$ $x(a-3)=2(a+1)(a-3)$

1. Fall: $a=3$ $\Rightarrow$ $0\cdot x=0$ wahr $\forall\, x\in\mathbb{Q}$

$L=\mathbb{Q}$

2. Fall: $a\neq 3$ $\Rightarrow$ $x(a-3)=2(a+1)(a-3)$ $\mid\div(a-3)$

$\Leftrightarrow$ $x=2(a+1)$

$\Rightarrow$ $L=\{2(a+1)\mid a\in IR\setminus\{-1;1;3\}\}$

e) $(x-c)(c+d)+(x-c-2d)(c-d)=2(d-c)(c-x) \quad , \; x\in\mathbb{Q} \;\wedge\; c,d\in\mathbb{Z}$

$$(x-c)(c+d)+(x-c-2d)(c-d)=2(d-c)(c-x)$$

$\Leftrightarrow \quad cx+dx-c^2-cd+cx-dx-c^2+cd-2cd+2d^2=2(cd-dx-c^2+cx)$

$\Leftrightarrow \quad 2cx-2c^2-2cd+2d^2=2cd-2dx-2c^2+2cx \quad |-2cx+2c^2-2cd$

$\Leftrightarrow \quad -4cd+2d^2=-2dx$

$\Leftrightarrow \quad -2d(2c-d)=-2dx$

1. Fall: $d=0 \;\Rightarrow\; 0=0\cdot x$ wahr $\forall\, x\in\mathbb{Q}$

$\Rightarrow \quad L=\mathbb{Q}$

2. Fall: $d\neq 0 \;\Rightarrow\; -2d(2c-d)=-2dx \quad |\div(-2d)$

$\Leftrightarrow \quad 2c-d=x$

$\Rightarrow \quad L=\{2c-d \mid d\in\mathbb{Z}\setminus\{0\}\wedge c\in\mathbb{Z}\}$

