Lösungen zu lineare Gleichungen mit Parameter:

Übungen:

a) $7a-3b+c-x=2b-3c$

b) $4a^2x-4ax=a-x$

c) $(x-m)(mx+2)+(1+mx)(2m-x)+m^4=1$

d) $\frac{2mx+n}{2m+n}+\frac{2m-nx}{2m-n}+\frac{2n^2(x+1)}{4m^2-n^2}=0 \quad mit\ n\neq\pm 2m$

zu a)

$$7a-3b+c-x=2b-3c \qquad |-7a+3b-c$$
$$\Leftrightarrow \quad -x=5b-7a-4c \qquad |\cdot(-1)$$
$$\Leftrightarrow \quad x=7a-5b+4c$$
$$\Rightarrow \quad L=\{7a-5b+4c\}\ \forall a,b,c\in IR$$

zu b)

$$4a^2x-4ax=a-x \qquad |+x$$
$$\Leftrightarrow \quad 4a^2x-4ax+x=a$$
$$\Leftrightarrow \quad x\left(4a^2-4a+1\right)=a$$
$$\Leftrightarrow \quad x(2a-1)^2=a$$

1. Fall: $(2a-1)^2\neq 0 \Leftrightarrow 2a-1\neq 0 \Leftrightarrow 2a\neq 1 \Leftrightarrow a\neq\frac{1}{2}$

$$\Rightarrow \quad x=\frac{a}{(2a-1)^2}$$
$$\Rightarrow \quad L=\left\{\frac{a}{(2a-1)^2}\right\}\ \forall a\in IR\setminus\left\{-\frac{1}{2}\right\}$$

2. Fall: $(2a-1)^2=0 \Leftrightarrow a=\frac{1}{2}$

$$\Rightarrow \quad x\cdot 0=-\frac{1}{2} \qquad Widerspruch\ \forall x\in IR$$
$$\Rightarrow \quad L=\{\ \}\quad für\ a=-\frac{1}{2}$$

zu c) $$(x-m)(mx+2)+(1+mx)(2m-x)+m^4=1$$

$$\Leftrightarrow \quad mx^2+2x-m^2x-2m+2m-x+2m^2x-mx^2=1$$

$$\Leftrightarrow \quad x+m^2x=1$$

$$\Leftrightarrow \quad x\left(1+m^2\right)=1 \qquad |\div\left(1+m^2\right)\neq 0\ \forall m\in IR$$

$$\Leftrightarrow \quad x=\frac{1}{1+m^2}$$

$$\Rightarrow \quad L=\left\{\frac{1}{1+m^2}\right\}\ \forall m\in IR$$

zu d) $$\frac{2mx+n}{2m+n}+\frac{2m-nx}{2m-n}+\frac{2n^2(x+1)}{4m^2-n^2}=0 \quad |\cdot(2m+n)(2m-n)$$

$$(2m-n)(2mx+n)+(2m+n)(2m-nx)+2n^2(x+1)=0$$

$$4m^2x+2mn-2mnx-n^2+4m^2-2mnx+2mn-n^2x+2n^2x+2n^2=0$$

$$4m^2x-4mnx+n^2x+4m^2+4mn+n^2=0 \qquad |-4m^2-4mn-n^2$$

$$4m^2x-4mnx+n^2x=-4m^2-4mn-n^2$$

$$x(4m^2-4mn+n^2)=-(4m^2+4mn+n^2)$$

$$x(2m-n)^2=-(2m+n)^2 \qquad |\div(2m-n)^2, da\ n\neq\pm 2m$$

$$x=-\frac{(2m+n)^2}{(2m-n)^2}$$

$$L=\left\{-\frac{(2m+n)^2}{(2m-n)^2}\right\}\ für\ n\neq\pm 2m$$

